**Advanced Quantum Theory**
WS 13/14

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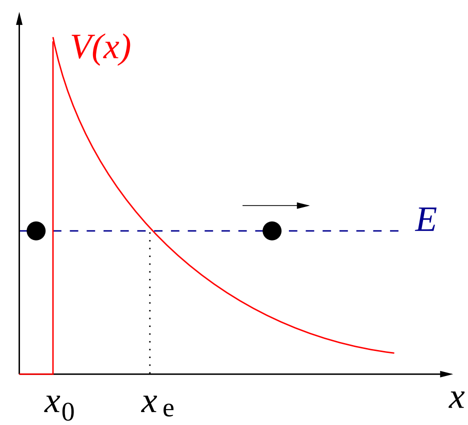
## Problem Set 1 (due Monday, 21.10.2013 in the lecture)

(1.1) $\alpha$-DECAY *(6 points)*

Consider an $\alpha$-particle of mass $m$ and charge $Z_\alpha e = 2e$, escaping from a nucleus whose charge drops down to $Ze$ during the process. The potential is modelled as shown below. For $x < x_0$ the attractive nuclear force dominates, creating a narrow potential well, while for $x > x_0$ the Coulomb repulsion $Z_\alpha Z e^2/x$ takes over.




(i) The semiclassical mean life time $\tau$ has been introduced in the lecture: $\tau = 1/R$, $R = (v/2x_0)\mathrm{e}^{-\gamma}$, $v = \sqrt{2E/m}$, $\gamma = (2/\hbar)\int_{x_0}^{x_\mathrm{e}} \sqrt{2m[V(x)-E]}\,\mathrm{d}x$. Show that for $x_\mathrm{e} \gg x_0$ one obtains

$$\gamma \simeq \pi \frac{\sqrt{2m} Z_\alpha e^2}{\hbar} \left( \frac{Z}{\sqrt{E}} - \frac{4}{\pi} \frac{\sqrt{Z x_0}}{e\sqrt{Z_\alpha}} \right).$$

(ii) Using the above results, estimate the mean life time of an $\alpha$-particle of kinetic energy 4.2 MeV, which tunnels out of a nucleus with final $Z = 90$. Assume $x_0 = 10^{-12}$ cm.

(iii)* [1]The decay law reads $N(t) = N(0)\mathrm{e}^{-t/\tau}$ so that the half-life time $T_{1/2} = (\ln 2)\tau = 0.693\,\tau$. Derive the TAAGEPERA-NURMIA formula

$$\log_{10} T_{1/2} = \alpha \left( \frac{Z}{\sqrt{E}} - Z^{2/3} \right) - \beta,$$

where $T_{1/2}$ is in years and $E$ in MeV, and $\alpha \simeq 2$ and $\beta \simeq 30$ are constants. *Hint:* Set $x_0 = \ell_0 Z^{1/3}$ (why?) with $\ell_0 \simeq 2 \times 10^{-15}$ m.

(1.2) VARIATIONAL APPROACH *(4 points)*

Pretend you do not know the ground state wave function of the H atom (do you?). Determine an upper bound for the ground state energy by using $\psi(r) = \mathrm{e}^{-\alpha r^2}$ as a trial function.

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[1] Starred problems (*) are voluntary but may bring you extra points.