Atoms and Clusters
WS 11/12

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Problem Set 1

(1.1) Wave nature of matter.

(i) Calculate the wavelength of a particle with mass $2\,\mathrm{mg}$ at a velocity of $1080\,\mathrm{km/h}$.

(ii) In an electron microscope the electrons are accelerated with $200\,\mathrm{kV}$. What is the wavelength of the electrons, and how large is their velocity? Is it necessary to consider relativistic motion?

(1.2) Photons. How many photons per second are radiated from an isotropically radiating, monochromatic source of power $1\,\mathrm{W}$ for the following wavelengths:

(i) $10\,\mathrm{m}$ radio wave,

(ii) $10\,\mathrm{cm}$ microwave,

(iii) $5890\,\mathrm{Å}$ yellow sodium line, and

(iv) $1\,\mathrm{Å}$ soft X-ray?

Calculate for each case the number of photons passing through an unit area normal to the direction of propagation at a distance of $10\,\mathrm{m}$ and the density of photons there.

(1.3) Minimum coupling. The Hamiltonian for a particle of mass m and charge q in an electromagnetic field

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \partial_t \boldsymbol{A}, \qquad \boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$$

has been introduced in the lecture and reads

$$H = \frac{p^2}{2m} - \frac{q}{2m}(\boldsymbol{A}\cdot\boldsymbol{p} + \boldsymbol{p}\cdot\boldsymbol{A}) + \frac{q^2}{2m}A^2 + q\phi.$$

(i) Classical motion: Show that the resulting equations of motion are equivalent to $m\ddot{\boldsymbol{r}} = \boldsymbol{F}$ where $\boldsymbol{F} = q(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$ is the Lorentz force.

(ii) Quantum dynamics: Show that the time-dependent Schrödinger equation

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(\boldsymbol{r},t) = H\Psi(\boldsymbol{r},t)$$

is invariant under the gauge transformation

$$\boldsymbol{A} = \boldsymbol{A}' + \boldsymbol{\nabla}\chi,$$

$$\phi = \phi' - \frac{\partial}{\partial t}\chi,$$

$$\Psi = \mathrm{e}^{\mathrm{i}q\chi/\hbar}\Psi'.$$